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NONLINEAR CONTROL IN THE SERVOMECANISMS FOR POSITIONING OF AN INDUSTRIAL ROBOT

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Abstract. The paper presents a methodology for simulation of a nonlinear servomechanism position control with or without the blocks for compensating of the hard nonlinearities.

Key words: robotics, linear control, hard nonlinearities and nonlinear control.

1. Introduction

Nonlinear control methodologies have applications in aircraft, spacecraft control, robotics, process control and biomedical engineering.

"Hand nonlinearities" are discontinuous nonlinearities and cannot be locally approximated by linear functions. These nonlinearities include: Coulomb friction, saturation, dead-zones (adhesion), backlash, hysteresis and are often found in control engineering domain. Their effects: instabilities, for example, cannot be derived from linear methods and nonlinear analysis techniques must be developed to predict a system's performance in the presence of these inherent nonlinearities and must be predicted or compensated.

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In this paper it is considered the simplest type of control strategy: independent joint control, where each axis of the manipulator is controlled as a single-input/single-output system (SISO) and any coupling effects due to the motion of the other links is treated as a disturbance.

The position control system is a system that converts a position input command to a position output response. A schematic layout of the servomotor and gear reduction is shown in Fig. 1.





Fig. 1 – Servo plant schematic (d.c. motor and gear reduction).

The electrical parameters are as follows:

 R_a – armature resistance, [Ω]; L_a – armature inductance, [H]; K_m – motor torque constant, [Nms/rad]; K_e – motor voltage constant, [V/rad/s].

The mechanical parameters are as follows:

 J_m – armature inertia, [Nms²]; J_s – load inertia, [Nms²]; B_m – armature frictional coefficient, [Nms/rad]; B_s – load frictional coefficient, [Nms/rad]; I – low gear ratio; M_r – resistant moment referred to the rotor shaft.

2. Linear Model

Equivalent inertia and friction referred to the rotor shaft (dynamic model) are:

$$J = J_m + J_s \frac{1}{i^2}; B = B_m + B_s \frac{1}{i^2}$$
(1)

18

From the Fig. 1, we can write the following equations based on Newton's law combined with Kirchhoff's law:

$$u_a = R_a i_a + L_a \frac{d i_a}{dt} + e \tag{2}$$

$$M_m - M_r = J \frac{d\omega_m}{dt} + B\omega_m \tag{3}$$

where: u_a armature voltage, $e = K_e \omega_m$ the motor e.m.f., ω_m armature speed, $M_m = K_m i_a$ motor torque, \dot{i}_a armature current and $K_e = K_m$ for permanent magnet d.c. motor.

Using Laplace Transforms the above equations can be expressed in terms of "s":

$$U_a(s) = (s L_a + R_a) I_a(s) + K_e \Omega_m(s)$$
⁽⁴⁾

$$K_m I_a(s) - M_r(s) = (sJ + B)\Omega_m(s)$$
⁽⁵⁾

and it results the scheme of preliminary linear model in Fig. 2, where:



Fig. 2 – Software Simulink/MATLAB for the nonlinear independent control position in an industrial robot(model of position controlled robot link driven by permanent magnet DC motor.

- Transf Fcn2 is the transfer function of d.c. motor:



- Transf Fcn 4 is the transfer function of mechanical transmission:

| | 1 |
|---|----------------|
| 6 | J + B |
| | Transfer fon 4 |

The constants for the conversion angle-voltage are: $K_i = K_T$ in the case of input θ_i or output (E-encoder) θ_s of the servomechanism. The scheme of servomechanism has a P-Controller with K_P -proportional gain.

The unit step, as:

$$\theta_i = \begin{cases} 0 & t = 0.5s(for example) \\ 1 & t > 0 \end{cases}$$

is widely used in studying input/output systems and step response of linear or nonlinear system is defined as the output $\theta_s(t)$ starting from initial condition.

3. Nonlinear Model

Saturation. Many manipulators incorporate current limiters (or maximum torque) in the servo-system to prevent damage that might result from overdrawing current.



Fig. 3 – The effects of saturation.

The effects of saturation are the decrease and delay of output θ_s . These effects are not compensated.

Friction. In the servomechanism position control - Fig. 1 exist: the motor brushes friction (dry friction: Coulomb and Stribeck friction), friction in the motor bearings, the gear friction (where friction is a function of the payload).

The LuGre dynamic model of friction with low speed is given by expression:

$$M_f = \lambda_0 z + \lambda_1 \frac{dz}{dt} + \alpha_2 \omega \tag{6}$$

$$\frac{d_z}{d_t} = \omega - \frac{\lambda_0 |\omega|}{g(\omega)}$$
(7)

$$g(\omega) = \alpha_0 + \alpha_1 e^{-\omega / \omega_{sk}}$$
(8)

where: α_0 - Coulomb friction; α_1 - Stribeck friction; α_2 - viscous friction; λ_0 - bristles stiffness; λ_1 - bristles damping; ω_{sk} - Stribeck velocity; ω - angular velocity.

This model has six parameters: α_0 , α_1 , α_2 , λ_0 , λ_1 and ω_{sk} which can be found experimentally. The bloc of LuGre model (4) is presented in Fig. 4 in Simulink/MATLAB.



Fig. 4 – LuGre friction model (Subsystem).

The solution of friction compensation are: 1 - use of lubrication to reduce friction; 2 - use of a dither signal (adding a high frequency signal to control signal who induces a mechanical vibration and prevents adhesion); 3 - acceleration feedback; 4 - model based friction compensation.

For the last solution it is considered the following adaptive scheme based on a Coulomb model.

$$M^* = a^* sign(\omega)$$
 (9)

$$\dot{z} = K u_c \operatorname{sign}(\omega) \tag{10}$$

$$a^{'} = z - K J \left| \omega \right| \tag{11}$$

where: K is an adaptation gain; J – inertia, z – a internal variable; M' – the estimate friction moment, $u_c(M_m)$ – the conventional controller output, ω – the relative velocity and a^* – the estimated Coulomb model parameter.

The block of adaptive friction model (5) is presented in Fig. 5 in Simulink.



Fig. 5 – Adaptive friction model (Subsystem 1).

The effect of friction is the existence of a considerable steady state error (Fig. 6 *a*). After friction compensation the steady state error approximately disappears, the settling time increases and the overshoot appears in the unit step response (Fig. 6 *b*).



Backlash, also known as play, is the difference 2D between tooth width and tooth space in mechanical gears or transmissions – Fig. 7. Backlash is a non-smooth nonlinear phenomenon, it is bad for control performance and it sometimes induces oscillations or steady state errors. The gears collision is due to the backlash and compliance of the gearbox.

The compensations of backlash are: 1 - mechanical solution; 2 - dual motor control; 3 - linear controller design; 4 - block of the backlash inverse.



Fig. 7 - Backlash model.

The third case is an advantageous method and the backlash can be compensated by introducing phase lead in the controller. For example, a lead compensator (filter) has the expression:

$$F(s) = \frac{T_1 s + 1}{T_2 s + 1} \tag{12}$$

where: T_1 , T_2 are constants.



Fig. 8 – Effect of backlash (a) and compensation of backlash (b).

For this example, the effect of backlash is the appearance of steady state error – Fig. 8 a and disappears in the presence of "lead compensator" – Fig. 8 b.



Fig. 9 – Sample step response of the linear (ideal) system and nonlinear compensated system in two cases of the proportional gain (P-Controller): a – proportional gain $K_p = 10$; b – proportional gain $K_p = 30$.

In the case of nonlinear (friction and backlash) compensated system with P-Controller results a diminution of settling time when the proportional gain increases, Fig. 9 *b*. In both examples in Fig. 9 the adaptive friction model (and Fig. 6 *b*) modifies simultaneously the transient response seeking the initial condition.

3. Conclusions

The utilization of software in Simulink facilitates the dynamic behaviour of a nonlinear closed-loop system. The "hard nonlinearities" produce, in general, in the unit step response: oscillations (instability) or steady state errors (lack of precision); these disadvantages can be removed, for example, with compensator blocks. In the case of a direct-drive robot with high-torque motors, the problems of backlash, friction and compliance due to the gears are eliminated. In exchange, the effect of the gear reduction is largely to decouple the system by reducing the inertia coupling among the joints.

REFERENCES

- Khalil H., Nonlinear Systems. Prentice-Hall (2002).
- Olson H., Friction Models and Friction Compensation. Journal of Control (1998).
- Receanu D., Budescu E., A New Method for the Dynamical Simulation of Mechanical System Using MATLAB-Simulink. International Conference on Computational
 @ Experimental Engineering and Sciences, Las Vegas-USA 29 March, 2010 ICCEES, 14, 1, 23–28 (2010).
- Schiffer J., *Dual Motor Control for Backlash Reduction*. Department of Automatic Control, Lund University (2009).

CONTROLUL NELINIAR ÎN SERVOMECANISMELE DE POZIȚIONARE ALE UNUI ROBOT INDUSTRIAL

(Rezumat)

Când se consideră liniare sistemele de control ale poziției se neglijează forțele și momentele neliniare asociate cu mișcarea elementelor cinematice componente ale robotului industrial. O astfel de aproximare se poate considera în cazul când robotul lucrează la viteze mici. La viteze mari de funcționare forțele neliniare Coriolis și centripete, proporționale cu pătratul vitezei, devin importante, iar modelul liniar cu o singură intrare/o singură ieșire(SISO) nu se poate aplica.

În acest caz se utilizează controlul multivariabil, când robotul este privit ca un sistem cu variabile multiple de intrare, respectiv variabile multiple de ieșire(MIMO) și când se compensează integral forțele neliniare din robot. În lucrare s-a considerat controlul independent al robotului, deci modelul SISO, la care se consideră și neliniaritățile de tipul: frecarea Coulombiană, saturația, zonele moarte, jocul și histerezisul (hard nonlinearities). Prin simulare s-au trasat graficele răspunsurilor la mărimea treaptă unitară de intrare, pentru comparație, în cazul servomecanismului de poziție ideal, cu neliniarități și în cazul compensării acestora. S-a constatat, astfel, necesitatea folosirii metodelor de compensare a efectelor produse de neliniarități pentru realizarea unei precizii adecvate și a stabilității dinamice în faza de regim permanent.

În cazul acționării robotului cu motoare cu moment mare fără reductor cu roți dințate, problemele jocului danturii, frecării, elasticității, date de roțile dințate, sunt eliminate. Însă avantajul transmisiei motor-reductor este acela că momentul de inerție redus la arborele motor este neinfluențat de momentul de inerție al sarcinii, datorită raportului de transmitere mare $(20 \div 200)$ al angrenajelor.