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COMPARATIVE STUDY OF THE EMPTYING PROCESS OF VARIABLE CROSS SECTION TANKS THROUGH AN EMPTYING ORIFICE

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Abstract. In this paper the emptying process of non-pressurized variable head tanks with variable cross-section area through an emptying small orifice will be investigated. Three types of tanks will be analyzed: the spherical tank, the V-shaped conical tank, and the A-shaped conical tank. This study will be performed comparatively with the cylindrical tank which will be considered as a reference tank. All tanks will have the same initial volume of the same liquid, the same initial head and an emptying orifice with the same characteristics, placed of the bottom side of the tank. The emptying process will be analyzed with a MATLAB/Simscape functional diagram which allows obtaining the numerical values of the most important emptying characteristic parameters.

Key words: variable cross-section area tank, emptying time, emptying orifice, functional diagram, MATLAB.

1. Introduction

During the working time of any kind of hydraulic system equipped with a tank, one of the principal problems is to control the liquid volume inside the tank, which depends of the supplying system, of the emptying system but also

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of the tank characteristics. For example, if the supplying system is switched off and the emptying process is based only on the hydrostatic liquid pressure, what is the influence of the cross-section tank on the emptying time of the tank? This can be reformulated as: how fast will be the emptying process of the same liquid volume under the same initial hydrostatic pressure contained in tanks with different cross-section? Moreover, what is the shape of the liquid level variation with respect of the time?

These are some of multiple aspects related with the emptying process of tanks which can be answered using, for example different MATLAB based methods. The method of numerical analysis procedures but also the Simscape functional diagram method for constant cross-section tanks with both constant and variable head are presented in (Zahariea, 2011a) for emptying orifice and (Zahariea, 2011b) for emptying pipe. The analytical method is based on the fundamental relationship for the tank emptying time. The Simscape functional diagrams method is based on the MATLAB/Simscape development methodology using Simscape functional blocks for simulate real physical elements, like: reservoir with constant head, reservoir with variable head, orifice with constant section, flow meter, etc. (Zahariea, 2010). The method of Simulink block diagrams (Dabney & Harman, 2004) for conical tanks with variable head is presented in (Zahariea, 2012).

According with the studies mentioned above, the method of Simscape functional diagrams should be used preferentially, especially for complex physical systems, because no process equations are required, but only a model file based on the functional diagram technique, which will automatically generate the characteristic equations system for the physical system analyzed, no matter how complex this system will be.

In this paper the emptying process of non-pressurized variable head tanks with variable cross-section through an emptying small orifice will be investigated. Three types of tanks will be analyzed: the spherical tank, the ∇ -shaped conical tank, and the \wedge -shaped conical tank. This study will be performed comparatively with the cylindrical tank which will be considered as a reference tank. All tanks will have the same initial volume of the same liquid, the same initial head and an emptying orifice with the same characteristics placed of the bottom side of the tank. The emptying process will be analyzed with a MATLAB/Simscape functional diagram which allows obtaining the numerical values of the most important emptying characteristic parameters

2. Geometric and Hydraulic Tanks Characteristics

Let us consider four tanks having the same liquid volume, $V_i=1 \text{ m}^3$ and the same initial head, H_i . All four tanks are filled with water at 20°C , with the density $\rho=998.465 \text{ kg/m}^3$ and the dynamic viscosity $\eta=1.0041 \cdot 10^{-3} \text{ Pa}\cdot\text{s}$.

The emptying process of all four tanks is developing using an emptying orifice placed of the bottom side of each tank with the same characteristics: diameter $D_o=10$ mm and flow coefficient $\mu_o=0.7$. All four tanks are non-pressurized (open tanks with atmospheric pressure at the free surface of the water inside the tanks) and the emptying process will be developed under the tank variable head.

Let us assume that the first tank has a circular constant cross-section with the initial head H_i , the constant diameter D_1 and the initial volume $V_i = V_{i1} = \pi D_1^2/4 \cdot H_i$, (Fig. 1). The cylindrical tank will be considered as the reference tank.

The second tank is a spherical tank with the initial head H_i , the radius $R_2 = H_i/2$ and the initial volume $V_i = V_{i2} = 4\pi/3 \cdot R_2^3 = \pi/6 \cdot H_i^3$, (Fig. 2). Considering that the initial liquid volume is $V_i=1$ m³, the initial head H_i can be expressed as follows:

$$H_i = \sqrt[3]{6/\pi} \quad (1)$$

As both the cylindrical and the spherical tanks have the same initial liquid volume $V_i = V_{i1} = V_{i2}$, the diameter D_1 can be expressed by:

$$D_1 = H_i \cdot \sqrt{2/3} \quad (2)$$

Assuming that z is the free liquid level position with $z \in [0, H_i]$, then the variable volume of the spherical tank $V_2(z)$ can be expressed by:

$$V_2(z) = \frac{\pi H_i^3}{6} \cdot \left(3 - 2 \frac{z}{H_i}\right) \cdot \left(\frac{z}{H_i}\right)^2 \quad (3)$$

The third tank is a ∇ -shaped conical tank with the initial head H_i , the maximum diameter D_3 placed on the upper side of the tank and the initial volume $V_i = V_{i3} = \pi/12 \cdot H_i D_3^2$, (Fig. 3). Considering that the $V_i = V_{i2} = V_{i3}$, the maximum diameter can be expressed by:

$$D_3 = H_i \cdot \sqrt{2} \quad (4)$$

The variable volume of the ∇ -shaped conical tank $V_3(z)$ can be expressed using the relationship:

$$V_3(z) = \frac{\pi H_i D_3^2}{12} \cdot \left(\frac{z}{H_i}\right)^3 \quad (5)$$

The fourth tank is a \blacktriangle -shaped conical tank with the initial head H_i , the maximum diameter D_4 placed on the bottom side of the tank and the initial volume $V_i = V_{i4} = \pi/12 \cdot H_i D_4^2$, (Fig. 4). Considering that $V_i = V_{i3} = V_{i4}$, the following relation can be obtained: $D_4 = D_3$.

The variable volume of the \blacktriangle -shaped conical tank $V_4(z)$ can be expressed by:

$$V_4(z) = \frac{\pi H_i D_3^2}{12} \cdot \left[1 - \left(1 - \frac{z}{H_i} \right)^3 \right] \quad (6)$$

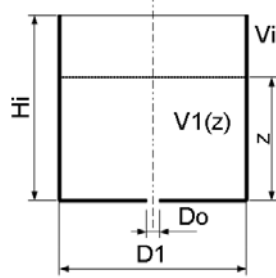


Fig. 1 – The cylindrical tank.

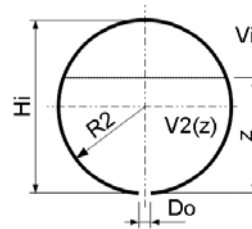


Fig. 2 – The spherical tank.

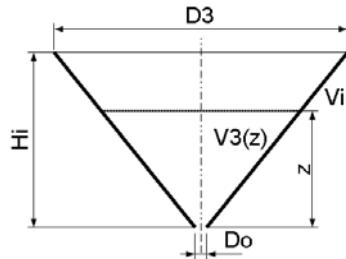


Fig. 3 – The V-shaped conical tank.

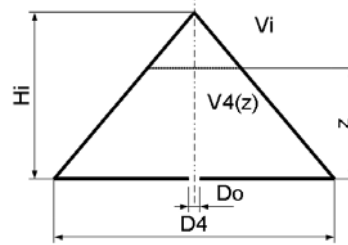


Fig. 4 – The A-shaped conical tank.

In Fig. 5 the dimensionless volume $\bar{V} = V/V_i \in [0,1]$ variation with respect to the dimensionless free liquid level position $\bar{z} = z/H_i \in [0,1]$ is comparatively presented for all four tanks.

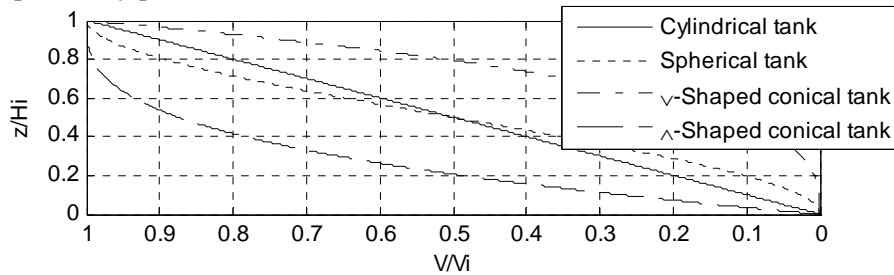


Fig. 5 – The level variation of the tanks liquid volume.

3. MATLAB/Simscape Functional Diagram

The MATLAB/Simscape functional diagram for modelling and simulating the emptying process of a tank through an orifice is presented in Fig. 6 (Zahariea, 2011a), (Zahariea, 2012).

The variable cross section tank with variable head is simulated with the “Variable Head Tank” functional element with a table-specified relationship between the water level z and the water volume inside the tank $V(z)$.

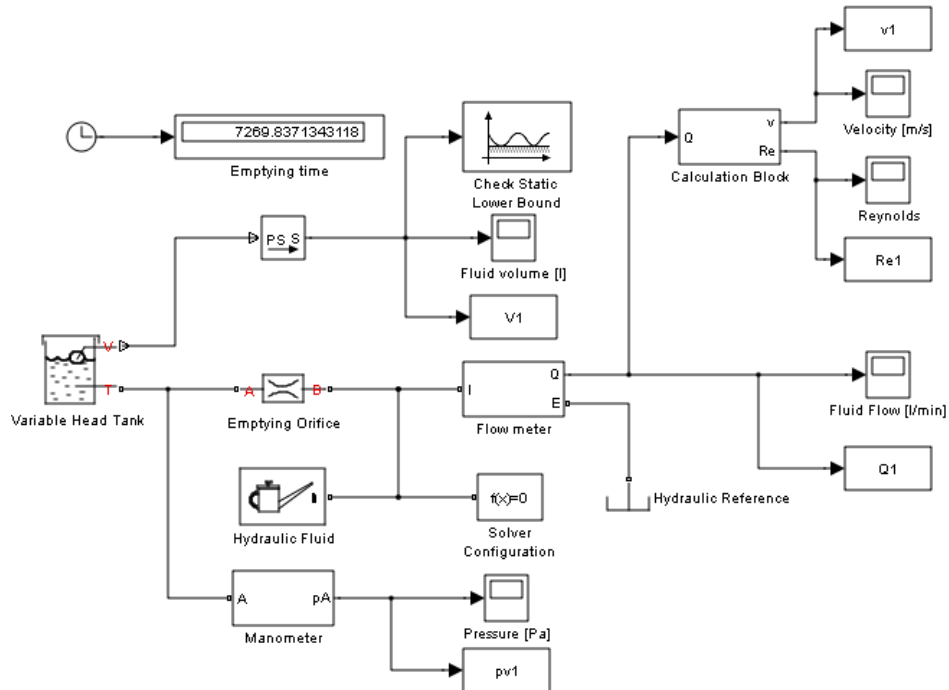


Fig. 6 – The functional diagram.

The tabulated fluid volumes and the tabulated fluid levels have been already defined in the MATLAB workspace using a script file (input.m) for initial data input. The emptying orifice is simulated with the “Emptying Orifice” functional element for which the orifice area $A_o = \pi D_o^2/4$ and the flow discharge coefficient μ_o are the main parameters. The working fluid is defined with the “Hydraulic Fluid” functional element, which is set for water at 20°C.

The “V” port of the “Variable Head Tank” functional element outputs the fluid volume of the tank through a PS-Simulink Converter which defines the fluid volume unit as liter. This signal is always compared with a reference value corresponding to the total emptying of the tank ($V_f=0$) in the “Check Static Lower Bound” block. When the fluid volume inside the tank will become equal with the reference value, the verification block stops the simulation and displays an error message. In this moment, the time simulation will be shown on the “Display” block named “Emptying time”. There are three subsystems: “Flow meter” for measuring the fluid flow; “Manometer” for measuring the pressure and “Calculation Block” for fluid velocity and Reynolds number computation.

During the simulation, there are some output parameters which will be saved back to the MATLAB workspace: the liquid volume inside the tank; the hydrostatic pressure at the bottom of the tank; the water flow, the velocity and

the Reynolds number of the fluid flow through the emptying orifice. For this purpose the Simulink block “To Workspace” has been used for each output parameter. The time variation of all these five output parameters can be observed directly from the functional diagram using the five “Scope” type blocks named: “Fluid volume [l]”, “Pressure [Pa]”, “Fluid flow [l/min]”, “Velocity [m/s]” and “Reynolds”. However, performing more detailed comparative graphic analysis can be achieved using the output parameters saved into the MATLAB workspace. These output parameters can be processed using another script file (output.m) for which the main tasks are to read the output parameters from MATLAB workspace and to generate the output comparative diagrams.

4. Numerical Results

Using the functional diagram and the working methodology describes above the following comparative results have been obtained: the dimensionless free liquid level position $\bar{z} = z/H_i \in [0,1]$ with respect to the emptying time (Fig. 7), the fluid flows with respect to the emptying time (Fig. 8), the fluid velocities with respect to the emptying time (Fig. 9), the fluid Reynolds number with respect to the emptying time (Fig. 10), the hydrostatic pressure at the bottom side of the tank with respect to the emptying time (Fig. 11).

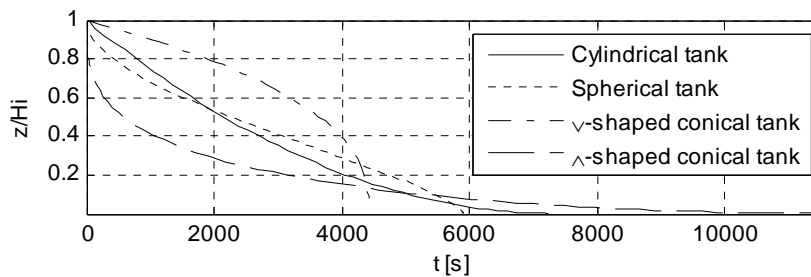


Fig. 7 – The time variation of the tanks dimensionless liquid level.

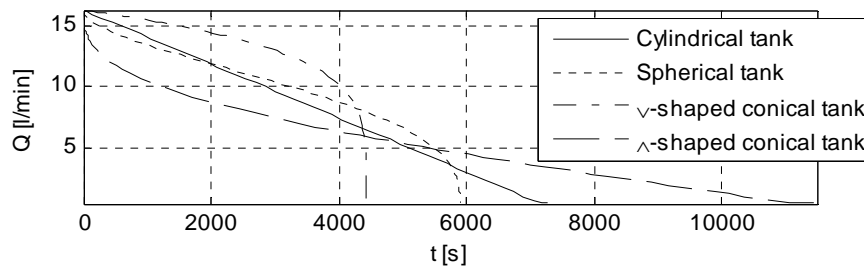


Fig. 8 – The time variation of the orifice fluid flows.

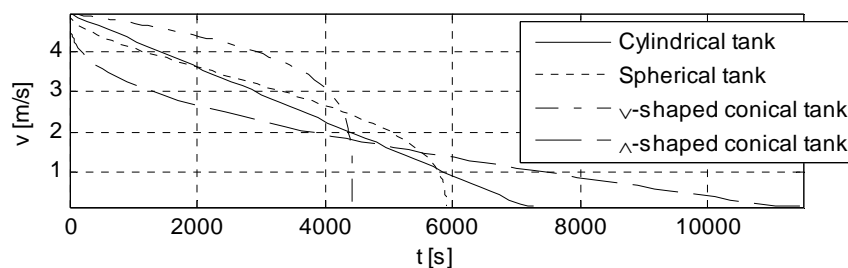


Fig. 9 – The time variation of the orifice fluid velocities.

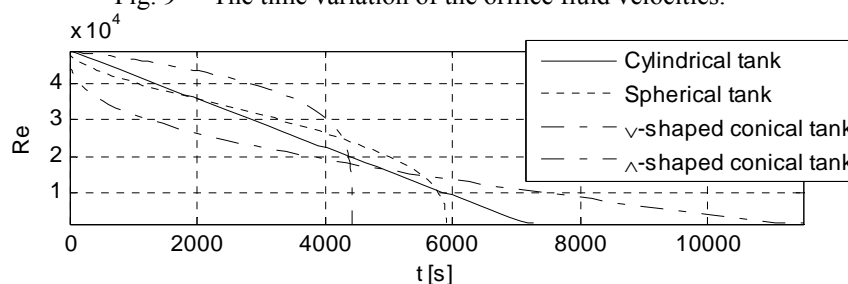


Fig. 10 – The time variation of the orifice fluid Reynolds numbers.

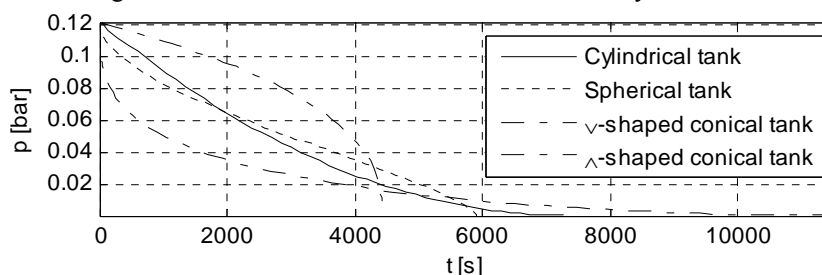


Fig. 11 – The time variation of the hydrostatic pressures.

5. Conclusions

With this analysis the following emptying times have been obtained: 7270 s for the cylindrical tank, 5901 s for the spherical tank, 4426 s for the ∇ -shaped conical tank and 11484 s for the \wedge -shaped conical tank, thus the fastest emptying process is obtained for the ∇ -shaped conical tank, while the slowest emptying process is obtained for the \wedge -shaped conical tank.

The MATLAB/Simscape functional diagram for modelling and simulating the emptying process of a tank has been presented in (Zahariea, 2011a) for constant cross-sectional area tanks and in (Zahariea, 2012) for variable cross-sectional area tanks. The first version has been based only on a single file (model.mdl) which was a Simscape model file while the second version has been based on an improved working technique with two files, one script file (input.m) for initial data input which generates in the MATLAB

workspace all the physical system variables and the second file (model.mdl) being a Simscape file which reads the workspace variable, simulates the emptying process and generates the output diagrams.

In this paper a more advanced working technique has been presented, with three files: the first file (input.m) is a script file for initial data input which generates in the MATLAB workspace all the physical system variables; the second file (model.mdl) is a Simscape model file which reads the workspace variable, simulates the emptying process and writes back into the MATLAB workspace the output parameters; the third file (output.m) is a script file which reads the output parameters from the MATLAB workspace and generates the output diagrams.

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STUDIUL COMPARATIV AL PROCESULUI DE GOLIRE A REZERVOARELOR CU SECȚIUNE VARIABILĂ PRINTR-UN ORIFICIU DE GOLIRE

(Rezumat)

În această lucrare se studiază procesul de golire a rezervoarelor deschise cu secțiune și sarcină variabile printr-un orificiu de golire plasat la baza rezervorului. Vor fi analizate trei tipuri de rezervoare: rezervorul sferic, rezervorul conic în formă de ∇ și rezervorul conic în formă de Δ . Se va efectua o analiză comparativă a celor trei tipuri de rezervoare în raport cu rezervorul cilindric care va fi considerat ca element de referință. Toate rezervoarele studiate au același volum inițial din același lichid de lucru, aceeași sarcină inițială și câte un orificiu de golire având aceleași caracteristici, plasat la baza rezervorului. Procesul de golire va fi analizat cu ajutorul unei scheme funcționale elaborată în limbajul de programare MATLAB/Simscape care permite obținerea valorilor numerice ale celor mai importanți parametri caracteristici ai procesului de golire.