BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 64 (68), Numărul 3, 2018
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

FROM SPIN TO HYPERBOLIC SKYRMIONS

BY

VLAD GHIZDOVĂŢ[*]

“Grigore T. Popa” University of Medicine and Pharmacy, Iaşi, Romania,
Faculty of Medicine, Biophysics and Medical Physics Department

Received: July 12, 2018
Accepted for publication: September 17, 2018

Abstract. In the context of a fractal framework of motion, a direct correspondence between the concept of spin and that of skyrmions is established by means of a specific mathematical model in the form of the Scale Relativity Theory.

Keywords: spin; skyrmions; fractal manifold.

1. Introduction

In spite of many significant shortcomings, the main scientific image of the atom was always, in the last hundred of years as well as today, the intuitive planetary one, amended perhaps with the idea that such a physical system is not plane but a spatial one – maybe spherical. This sphericity of the model, if real, should be due either to the non-centrality of forces dynamically responsible for determining the Kepler motion or, as we shall endeavor to explain here in the limits of Newtonian theory of forces, to the space expanse of matter, at the electronic and nuclear level. If this is the case, then the region of the nucleus can be characterized by a three-dimensional hyperbolic space related to the regular Euclidean space of our experience by harmonic mappings (Mazilu and Agop, 2012). With this statement we enter the realm of recent date of the hyperbolic skyrmions (Atiyah and Sutcliffe, 2004).

[*]Corresponding author; *e-mail*: vlad.ghizdovat@gmail.com

Indeed, one of the dominant contemporary concepts in the theory of structure of the nuclear matter, is that of skyrmion, which is but a variation on the subject of harmonic applications. The skyrmion is a topological soliton that can represent, in appropriate conditions, the nucleons, as forms of the nuclear matter. The history of this subject begins with the physicist Tony Skyrme, and the reader in need of following it closely can start with the very recollections of Skyrme on the beginnings (Skyrme, 1988), where the reasons, and the original works, are to be found. The main mathematical point of Skyrme's idea though, is a certain – 'almost harmonic' we should say – map from the usual space to the sphere of the relativistic space, built after the traditional manner of the variational problem leading to the Laplace and Schrödinger equations. Let us briefly describe this manner, following, by and large, the guidance of the recent work of Slobodeanu (Slobodeanu, 2009), however with the focus on maps from the usual space to the hyperbolic space, or hyperbolic plane. The reason is quite clear: based on our previous considerations, we feel that such maps legitimately describe the nuclear matter filling the central space of a classical atom. The 'filling' itself, we are talking about here, is actually represented, as we have seen before, as a harmonic map, or a variation thereupon. In such context, in the present paper a correspondence between the concepts of spin and skyrmions is established.

2. Mathematical Model

If, through an extended inertia principle, we assign masses to forces, it results that these masses cannot be tensors (as is the classical case of the electromagnetic mass), they must be coefficients which connect the forces with the spin properties of matter; these properties can be defined only in an extended particle model. Simply said, masses are three coefficients which, from a geometrical point of view, must be coordinates in a three-dimensional Lorentz geometry. The entire theory of forces is thus a gauge theory based on the spin properties of matter, which we consider to be fundamental.

Let us note that both the spin and the isospin of elemental particles are physical quantities usually represented by the Hermitian matrix (Schwartz, 1977).

$$Q = \begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & -\cos\theta \end{pmatrix} \tag{1}$$

$$Q^{+} = Q \tag{2}$$

where θ and ϕ are real variables. Noted here with „+”, the Hermitian transpose, Q^{+}, firstly imposes a transpose of the matrix and after a complex

conjugate of its elements. The Q matrix has eigenvalues of ± 1 and, up to a common factor with a kinetic moment dimension, it specifies
the spin or isospin of elemental particles. The general terms definition of such a Hermitian matrix does not require special meanings for θ and ϕ, excepting the fact that they must be real.

Although, if we accept the idea that any physical quantities which involves spatial extension must always have two real and distinct eigenvalues in any direction in space and that, in the case of an isotropic distribution, they must not depend on directions, then θ and ϕ will be identified with the usual angles on the unit sphere.

In this context, any physical quantity which involves spatial extension definition will be based on the Hermitian matrix M (Mazilu and Agop, 2010; Mazilu and Agop, 2012).

$$M = \lambda E + \mu Q \tag{3}$$

$$M^{+} = M \tag{4}$$

with λ and μ real and E the unit matrix. The eigenvalues of this matrix are $\lambda \pm \mu$ and they are not dependent on the spatial directions, given by the θ and ϕ angles. Considering the fact that the Q matrix is involutive, because it has a null trace, the Hamilton-Cayley equations allows us to rewrite the M matrix as (Mazilu and Agop, 2010; Mazilu and Agop, 2012):

$$M = \rho \exp(\psi Q) \tag{5}$$

$$\rho^2 = \mu^2 - \lambda^2 \tag{6}$$

$$\tanh \psi = \frac{\mu}{\lambda} \tag{7}$$

Such a representation is obviously valid as long as λ and μ meet the condition:

$$-1 < \frac{\mu}{\pi} < 1 \tag{8}$$

Indeed, for condition (8) Eq. (7) makes sense for any ψ .

Now, if M is a distance, relations (5)-(7) define the original ansatz of Skyrme (Skyrme, 1962), which gives the solution for the Skyrme nonlinear equations in the form of the „hedgehog" skyrmion. This solution does not represent Euclidian skyrmions, but hyperbolic ones. Otherwise, it is shown in (Atiyah and Sutcliffe, 2004) that Euclidian skyrmions, with pions having mass, lead to almost similar detailed results with the hyperbolic skyrmions, with zero mass pions.

We must note that, in the unification theories, the presence of skyrmions is mandatory, considering the fact that they are responsible for charges quantization (for details see both Skyrme's work (Skyrme, 1962) and, recently (Kanasz-Nagy *et al*., 2015)).

One of the advantages of a gauge theory, as presented in this work, is its versatility: it can be used in a stochastic approach of interaction, which is particularly attractive, for example, in plasma or nanostructures theories, just to name two high interest research fields. Moreover, the explicit connection of force with the boundary limit of an extended particles, limit which is "imposed" by the spin structure and thus, by the path integral (Feynman and Hibbs, 1965), with specific dynamics on continuous and non-differentiable curves (Peano curves in $D_F = 2$ fractal dimension at a Compton scale resolution) allows us to say that a fractal theory through "holographic" interactions seems more adequate in this case. Indeed, the fractal approach through the Scale Relativity Theory is based on differential equations with partial derivatives (Nottale, 2011; Mercheș and Agop, 2016). Thus, it can be shown that a wave theory is crucial in applying fractals to physical and technological problems (Agape *et al*., 2016; Agape *et al*., 2017; Gaiginschi and Agape, 2016; Gaiginschi *et al*., 2011; Gaiginschi *et al*., 2014a; Gaiginschi *et al*., 2014b; Gaiginschi *et al*., 2017; Vornicu *et al*., 2017) which involve material structures (Schwartz, 1977).

3. Conclusions

A specific mathematical model was developed with the aim to establish a conection betweem the concept of spin and that of skyrmion.

REFERENCES

Agape I., Gaiginschi L., Gaiginschi R., Baltagiu D., *Upon a Simple Method for Generating Transversal Profile of the Volumic Blower Rotor Used in Supercharging ICE*, Ingineria Automobilului, **38**, 22-25 (2016).

Agape I., Gaiginschi L., Ianuş G., Cimpoeşu N., *The Analysis of an Internal Combustion Engine Breakdown - Case Study*, IOP Conference Series: Materials Science and Engineering, **209**, *1*, 012077 (2017).

Atiyah M.F., Sutcliffe P., *Skyrmions, Instantons, Mass and Curvature*, arxiv:hep-th/0411052v1 (2004).

Feynman R.P., Hibbs A.R., *Quantum Mechanics and Path Integrals*, McGraw Hill, New York, 1965.

Gaiginschi L., Agape I., *Upon an Issue of Correlation Between the Running Speed of Vehicles and Traffic Capacity of a Road Section*, IOP Conference Series: Materials Science and Engineering, **147**, *1*, 012118 doi:10.1088/1757-899X/147/1/012118 (2016).

Gaiginschi L., Gaiginschi R., Agape I., Hantoiu V., *Researches Regarding Impact Energy Dissipation by breaking for Front – Rear Collisions*, Ingineria Automobilului, **18**, 20-23 (2011).

Gaiginschi L., Agape I., Sachelarie A., Girbaci M.A., *Virtual High Speed Diesel Engine – A Simulated Experiment, Part I: Combustion Dynamics*, Applied Mechanics and Materials, **659**, 189-194 (2014a).

Gaiginschi L., Agape I., Sachelarie A., Girbaci M.A., *Virtual High Speed Diesel Engine – A Simulated Experiment. Part II: The Influence of Polytropic Exponent upon Combustion Product Formation,* Applied Mechanics and Materials, **659**, 195-200 (2014b).

Gaiginschi L., Agape I., Talif S., *Upon the Reconstruction of Accidents Triggered by Tire Explosion. Analytical Model and Case Study*, IOP Conf. Series: Materials Science and Engineering, **252,** 012016 doi:10.1088/1757-899X/252/1/012016 (2017).

Kanasz-Nagy M., Dora B., Demler E.A., Zarane G., *Stabilizing the False Vacuum. Mott Skyrmions*, Nature Scientific Reports, **5**, 7692 (2015).

Mazilu N., Agop M., *La răscrucea teoriilor. Între Newton și Einstein*. *Universul Barbilian*, Ars Longa, Iași, 2010.

Mazilu N., Agop M., *Skyrmions – a Great Finishing Touch to Classical Newtonian Philosophy*, Nova Publishers, New York, 2012.

Mercheş I., Agop M., *Differentiability and Fractality in Dynamics of Physical Systems*, World Scientific Publisher, Singapore, 2016.

Nottale L., *Scale Relativity and Fractal Space-Time: A New Approach to Unifying Relativity and Quantum Mechanics*, Imperial College Press, London, 2011.

Schwartz M., *An Axiomatic Deduction of the Pauli Spin Theory*, International Journal of Theoretical Physics, **16**, 249-257 (1977).

Skyrme T.H.R., *A Unified Field Theory of Mesons and Barions*, Nuclear Physics, **31**, 556-569 (1962).

Skyrme T.H.R., *The Origins of Skyrmions*, International Journal of Modern Physics A, **3**, 2745-2751 (1988).

Slobodeanu R., *On the Geometrized Skyrme and Faddeev Models*, arXiv:0809.4864v5 [math.DG] (2009).

Vornicu V., Ulian T., Rakosi E., Manolache Gh., Gaiginschi L., *Theoretical Model for Determination of the Spark Ignition Engine Thermo-Gasodynamic Parameters on Various Functional Conditions*, IOP Conference Series: Materials Science and Engineering, **252,** *1*, 012073 doi:10.1088/1757-899X/252/1/012073 (2017).

DE LA SPIN LA SKYRMIONI HIPERBOLICI

(Rezumat)

În contextul teoriei fractale a mișcării, o corelație între conceptul de spin și cel de skyrmion este stabilită prin intermediul teoriei relativității de scală.