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ON THE COHERENCE IN THE BIOLOGICAL STRUCTURES BY MEANS OF A "HIDDEN SYMMETRY"

BY

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Abstract. In the framework of the Scale Relativity Theory with arbitrary constant fractal dimension, the coherence of the entities of all biological structure is analyzed. The presence of a "hidden symmetry" of the dynamic equations implies both the explicitation of the synchronization group and of the synchronization of the field equations. Since the solutions of synchronization field equations is reduced to that of the Laplace equation in regular space an intimate interrelationship between gravitation and light is revealed.

Keywords: coherence; Scale Relativity Theory; "hidden symmetry"; Lie groups.

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1. Introduction

Biological systems can be assimilated to complex systems taking into account their structural-functional duality (Badii and Politi, 1997). The standard models used to study the dynamics of the all biological system are based on the hypothesis, otherwise unjustified, on the differentiability of the variables that describes it. The success of the differentiable models must be understood sequentially, *i.e.* there are domains large enough for the differentiability to be valid.

But differential methods fail when facing the reality: instabilities of the biological systems dynamics that can generate chaos or patterns through self-structuring, must be analyzed by means of the non-differentiable methods (Mercheş and Agop, 2016; Nottale, 2011).

In the present paper, using the non-differentiable mathematical procedures in the form of Scale Relativity Theory with arbitrary constant fractal dimension (Mercheş and Agop, 2016), coherence in biological system is analyzed.

2. The "Hidden Symmetry"

Let us consider the one- dimensional equation of the harmonic oscillator from the Fractal Mechanics (Mercheș and Agop, 2016) in the form of Scale Relativity with arbitrary constant fractal dimension:

$$\frac{d^2 X}{dt^2} + \Omega^2 X = 0 \tag{1}$$

with

$$\Omega^2 = \frac{(\pi\varepsilon)^2}{m_0^2 (dt)^{(4/D_F)-2}} \tag{2}$$

where X is the fractal spatial coordinate, t is the non-fractal time with functionality of affine parameter of motion curve, ε is the fractal energy of the microparticle, m_0 is the rest mass of the microparticle, λ is a coefficient associated to the fractal – non-fractal transition, dt is the scale resolution and D_F is the fractal dimension of the motion curve. A such differential equation can describe various behaviors of biological structures (Badii and Politi, 1997; Mazilu and Agop, 2012).

The general solution of the equations is writing in the form:

$$X(t) = z e^{i(\Omega t + \theta)} + \bar{z} e^{-i(\Omega t + \theta)} \tag{3}$$

where z is complex amplitude, $\bar{z}$ is the complex conjugate of z and θ is a specific phase.

Thus, z, $\bar{z}$ and θ label each microparticle from an eventual system that has a general characteristic the equation of state (1) and, consequently, the same Ω.

In such conjecture, can a priori be established a connection between the microparticles of the system. Since (1) has “hidden symmetry” in the form of homographic group, we can answer to this question positively.

Indeed, the ratio of two independent linear solutions of Eq. (1), τ, is a solution of Schwartz's differential equation (Mihăileanu, 1971):

$$\{\tau, t\} = \frac{d}{dt}\left(\frac{\tau''}{\tau'}\right) - \frac{1}{2}\left(\frac{\tau''}{\tau'}\right)^2 = 2\Omega^2, \qquad \tau' = \frac{d\tau}{dt}, \qquad \tau'' = \frac{d^2\tau}{dt^2} \tag{4}$$

The left part of (4) is invariant with respect to the homographic transformation:

$$\tau \leftrightarrow \tau' = \frac{a_1\tau + b_1}{c_1\tau + d_1} \tag{5}$$

with a_1, b_1, c_1, d_1 real parameters. The set of all transformations (4) corresponding to all possible values of these parameters is the group SL(2R).

Thus, the system of all the microparticles having the same Ω is in the bi-univocal correspondence with the transformations of the group SL(2R). This allows the construction of a “personal” parameter τ for each microparticle of the system separately. Indeed, we choose as “guide” the general form of the solution of the Eq. (4) which is writing as

$$\tau' = u + v\tan(\Omega t + \theta) \tag{6}$$

where u, v and θ are constants, and characterizes a microparticle of the system.

By identifying the phase from (6) with one from (3), we can write the “personal” parameter of the microparticle as:

$$\tau' = \frac{z + \bar{z}\tau}{1 + \tau}, \qquad z = u + iv, \qquad \bar{z} = u - iv, \qquad \tau \equiv e^{2i(\Omega t + \theta)} \tag{7}$$

The fact that (7) is also a solution of the Eq. (4) implies, by expliciting of (3), the group of transformations (Mazilu and Agop, 2012):

$$\begin{aligned} z' &= \frac{a_1 z + b_1}{c_1 z + d_1} \\ k' &= \frac{a_1\bar{z} + d_1}{c_1 z + d_1}k \end{aligned} \tag{8}$$

The infinitesimal generators of the group (8)

$$B_1 = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}} \ , \qquad B_2 = z\frac{\partial}{\partial z} - \bar{z}\frac{\partial}{\partial \bar{z}} \ ,$$
$$B_3 = z^2\frac{\partial}{\partial z} + \bar{z}^2\frac{\partial}{\partial \bar{z}} + (z-\bar{z})k\frac{\partial}{\partial k} \tag{9}$$

with commutation relations:

$$[B_1, B_2] = B_{1,}[B_2, B_3] = B_3, \qquad [B_3, B_1] = -2B_2 \tag{10}$$

reveals the same structure as the Lie algebra of the group SL(2R).

3. The "Qualities" of the Synchronization Group

Its structures is given by Eq. (10) and consequently, the structure constants are

$$C_{12}^1 = C_{23}^3 = -1, \ \ C_{31}^2 = -2 \tag{11}$$

the rest of the them being zero. Therefore, the invariant quadratic form is given by the "quadratic" tensor of the group (8), that is:

$$C_{\alpha\beta} = C_{\alpha\nu}^{\mu} C_{\beta\mu}^{\nu} \tag{12}$$

where summation over repeated indices is understood.

Using (11) and (12), the tensor $C_{\alpha\beta}$ writes

$$C_{\alpha\beta} = \begin{pmatrix} 0 & 0 & -4 \\ 0 & 2 & 0 \\ -4 & 0 & 0 \end{pmatrix} \tag{13}$$

meaning that the invariant metric of the group has the form:

$$\frac{ds^2}{g} = \Omega_0^2 - 4\Omega_1\Omega_2 \tag{14}$$

where g is an arbitrary constant factor and Ω_α three differential 1-forms, absolutely invariant through the group (8).

Barbilian (Mazilu and Agop, 2012) takes these 1-forms as being given by:

$$\Omega_0 = i\left(\frac{dk}{k} - \frac{dz + d\bar{z}}{z - \bar{z}}\right) \tag{15}$$

$$\Omega_1 = \bar{\Omega}_2 = \frac{d\bar{z}}{k(z - \bar{z})} \tag{16}$$

and the metric becomes:

$$\frac{ds^2}{g} = -\left(\frac{dk}{k} - \frac{dz + d\bar{z}}{z - \bar{z}}\right)^2 + 4\frac{dzd\bar{z}}{(z - \bar{z})^2} \tag{17}$$

Is the worthwhile to mention a property connected to the integral geometry: the “Barbilian” group (8) is measurable.

Indeed, it is simply transitive and, since its structure vector:

$$C_\alpha = C^\nu_{\nu\alpha} \tag{18}$$

is identical null, as it can be seen from (12), this means that is possess an invariant function given by:

$$F(z, \bar{z}, k) = -\frac{1}{(z - \bar{z})^2 k} \tag{19}$$

As a result, in space of the field variables $(z, \bar{z}, k)$ can a priori be construct a probabilistic theory (Jaynes, 1973) based on the elementary probability:

$$dP(z, \bar{z}, k) = -\frac{dz\Lambda d\bar{z}dk}{(z - \bar{z})^2 k} \tag{20}$$

where Λ denotes the exterior product of the 1-forms.

Moreover, for $\Omega_0 = 0$ a parallel transport on Lobacewski plane is given. Indeed, by means of the usual relations (see also the relations (7))

$$z = u + iv, \qquad k = e^{i\theta} \tag{21}$$

Ω_0 takes the form:

$$\Omega_0 = -\left(d\theta + \frac{du}{v}\right) \tag{22}$$

and therefore, the restriction $\Omega_0 = 0$ becomes:

$$d\theta = -\frac{du}{v} \tag{23}$$

which is the definition of the Levi-Civita parallelism of angle.

In this case the metric (17) takes the form:

$$ds^2 = \frac{dzd\bar{z}}{(z-\bar{z})^2} = \frac{du^2 + dv^2}{v^2} \tag{24}$$

i.e. the metric of Lobacewscki plane in Poincare sense (Mazilu and Agop, 2012).

4. Synchronization Field Equations

Using the variational principle Matzner-Misner (Mazilu and Agop, 2012) applied to the Lagrangean:

$$L = 4\frac{\nabla z \nabla \bar{z}}{(z-\bar{z})^2} - \left(\frac{\nabla k}{k} - \frac{\nabla z + \nabla \bar{z}}{z-\bar{z}}\right)^2 \tag{25}$$

the synchronization field equations in the coordinates (u, v, θ) become:

$$\begin{gathered}
\Delta\theta + \frac{1}{v}\Delta u - \frac{1}{v^2}\nabla u \nabla v = 0 \\
\Delta\theta - \frac{1}{v}\nabla\theta\nabla u = 0 \\
\Delta v - \frac{1}{v}(\nabla v)^2 - \nabla u \nabla \theta = 0
\end{gathered} \tag{26}$$

Transferring the differential condition $\Omega_0 = 0$ in operational condition:

$$\nabla\theta + \frac{\nabla u}{v} = 0 \tag{27}$$

the first Eq. (26) in the form:

$$\nabla\left(\nabla\theta + \frac{\nabla u}{v}\right) = 0 \tag{28}$$

is identically satisfied. Then, the second and the third Eqs. (26) become:

$$\Delta u = -2\nabla u \frac{\nabla v}{v}$$
$$\Delta v = \frac{(\nabla u)^2 - (\nabla v)^2}{v} \tag{29}$$

Using the complex quantities z and $\bar{z}$ given by (7) the above equations take the simple forms:

$$(z - \bar{z})\Delta z = 2\nabla z \nabla z$$
$$(z - \bar{z})\Delta \bar{z} = 2\nabla \bar{z} \nabla \bar{z} \tag{30}$$

The general solutions of these equations are the expressions:

$$z = -i\frac{\cosh \psi - e^{-i\alpha} \sinh \psi}{\cosh \psi + e^{-i\alpha} \sinh \psi}$$
$$\bar{z} = i\frac{\cosh \psi - e^{i\alpha} \sinh \psi}{\cosh \psi + e^{i\alpha} \sinh \psi} \tag{31}$$

with

$$\Delta \Psi = 0$$

and α real. Thus, here the solutions of synchronization field equations is reduced to that of the Laplace equation in regular space. However, by (31) the synchronization potential has a particular meaning: it is either an asymptotic direction for a Kepler- type motion, or one such direction in a classical description of light. Therefore, our models reveal an intimate interrelationship between gravitation and light.

5. Conclusions

The main conclusions of the present paper are the followings:

i) Using a one-dimensional differential equation of the harmonic oscillator from the Fractal Mechanics in the form a Scale Relativity with arbitrary constant fractal dimension, various behaviors of biological structures are analyzed. In particular case, the coherence between the entities of a complex system is established;

ii) a "hidden symmetry" in the form of Barbilian's group assimilated to a synchronization group is given. Then some "qualities" of this group is obtained (the structure of the group, the invariant metric, the invariant function, the Levi-Civita parallelism angle, etc.);

iii) The synchronization field equations are obtained. The solutions of these equations in the particular case of a Levi-Civita parallelism of angle reveale an intimate interrelationship between gravitation and light.

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ASUPRA COERENŢEI ÎN STRUCTURILE BIOLOGICE PRINTR-O "SIMETRIE ASCUNSĂ"

(Rezumat)

Utilizând Teoria Relativităţii de Scară în dimensiunea fractală constant arbitrară se analizează coerenţa entităţilor în structurile biologice. Existenţa unei "simetrii ascunse" a ecuaţiilor de dinamică implică atât explicitarea grupului de sincronizare cât şi a câmpurilor prin care aceasta se realizează. Întrucât soluţiile câmpurilor de sincronizare sunt reductibile la cele ale unei ecuaţii Laplace în spaţiul regular, atunci între gravitaţie şi lumină se relevă o legătură fundamentală.