BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 66 (70), Numărul 2, 2020
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

DISPERSIVE BEHAVIORS VIA NON-DIFFERENTIABILITY IN COMPLEX FLUIDS

BY

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Received: May 26, 2020
Accepted for publication: June 28, 2020

Abstract. Assuming that the motions of a complex fluid structural units take place on fractal curves, non-linear dispersive-type effects are analyzed. It results that the transport phenomena in complex fluids are dictated by cnoidal oscillation modes, their degeneration implying either periodic-type behaviors, quasi-periodic-type behaviors, or solitonic-type behaviors. All of these show the complexity of interactions taking place between the complex fluid entities.

Keywords: dispersive behaviors; complex fluid; non-differentiability; scale relativity.

1. Introduction

The most important aspect of complex systems is that the overall interactions of the structural units can dictate the functionalities of the whole systems, which individually does not present itself. Complex systems require a

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substantial energy to sustain their structural and functional behaviors. Even some small differences in one component can have significant consequences for the evolution of the whole system. Natural complex systems often show a high level of robustness due to redundancy in their components and interactions. Complex subsystems often combine to create new levels of functionalities. Probably the most noteworthy features of complex systems is that the evolutions observed in different systems can be defined by the same fundamental theory. For instance, the analytic gas reaction on a platinum surface (Rotermund *et al*., 1990), the aggregation of social amoebae (Foerster *et al*., 1988), or the cardiac fibrillation that leads to sudden death (Fenton *et al*., 2009) are all well defined by the same theoretical model through similar equations. Mathematical models, and new theories need to be further developed in such a way that in the end we are able to work with systems that are simultaneous sufficiently abstract and detailed so they can be implemented to the vast range of natural or artificial systems. These models need to be able to described the evolution of the systems at different temporal or spatial scales, the self-organization processes different sub systems or of the whole complex system, the implications of different the individual histories describing each structural unit and the global features of the systems, the appearance of phenomena induced by the interactions at different resolutions scales etc. Understanding and attempting to control the functional, structural, and dynamical properties of complex systems and utilizing them for the unravel of basic physical process or the complex behavior of malignant cells can become building blocks for the developing of new directions in theoretical physics.

2. Methods

Let us consider the geodesics equation on a fractal manifold free of any constraint (Tesloianu *et al*., 2015). In such conjecture, separating the real part from the imaginary one of the velocity field, at the differentiable resolution scale, we obtain:

$$\frac{\hat{d}\mathbf{V}_D}{dt} = \frac{\partial \mathbf{V}_D}{\partial t} + (\mathbf{V}_D \cdot \nabla)\mathbf{V}_D - (\mathbf{V}_F \cdot \nabla)\mathbf{V}_F - \lambda\, (dt)^{\left(2/D_F\right)-1} \Delta \mathbf{V}_F + \\ + \frac{\sqrt{2}}{3}\lambda^{3/2} (dt)^{\left(3/D_F\right)-1} \nabla^3 \mathbf{V}_D = 0 \quad (1)$$

and, at the fractal resolution scale:

$$\frac{\hat{d}\mathbf{V}_F}{dt} = \frac{\partial \mathbf{V}_F}{\partial t} + (\mathbf{V}_F \cdot \nabla)\mathbf{V}_D - (\mathbf{V}_D \cdot \nabla)\mathbf{V}_F - \\ -\lambda (dt)^{\left(2/D_F\right)-1} \Delta \mathbf{V}_D + \frac{\sqrt{2}}{3}\lambda^{3/2} (dt)^{\left(3/D_F\right)-1} \nabla^3 \mathbf{V}_F = 0 \quad (2)$$

The quantities from (1) and (2) are defined in (Tesloianu *et al*., 2015).
For irrotational motions

$$\nabla\times\hat{\boldsymbol{V}}=0,\nabla\times\boldsymbol{V}_D=0,\nabla\times\boldsymbol{V}_F=0 \tag{3}$$

the velocity field takes the form

$$\hat{\boldsymbol{V}}=-2i\lambda\left(dt\right)^{(2/D_F)-1}\nabla\ln\psi \tag{4}$$

or, explicitly, with $\psi=\sqrt{\rho}\exp\left(iS\right)$,

$$\begin{gathered}\hat{\boldsymbol{V}}=2\lambda dt^{(2/D_F)-1}\nabla S-i2\lambda\left(dt\right)^{(2/D_F)-1}\nabla\ln\rho\\ \boldsymbol{V}_D=2\lambda\left(dt\right)^{(2/D_F)-1}\nabla S\\ \boldsymbol{V}_F=2\lambda\left(dt\right)^{(2/D_F)-1}\nabla\ln\rho\end{gathered} \tag{5}$$

where $\sqrt{\rho}$ is an amplitude and S a phase.

In such a context, the fractal fluid is incompressible, *i.e.* $\rho=\text{const.}$, and thus Eqs. (1) and (2) take the single form

$$\frac{\hat{d}\boldsymbol{V}_D}{dt}=\frac{\partial\boldsymbol{V}_D}{\partial t}+\left(\boldsymbol{V}_D\cdot\nabla\right)\boldsymbol{V}_D+\frac{\sqrt{2}}{3}\lambda^{3/2}\left(dt\right)^{(3/D_F)-1}\nabla^3\boldsymbol{V}_D=0 \tag{6}$$

An explicit form of the velocity field $\boldsymbol{V}_D$ is obtained for the one-dimensional case. In dimensionless variables

$$\omega t=\tau_1,kx=\xi_1,\theta=\xi_1-M\tau_1,\frac{V_D}{c}=\Phi \tag{7}$$

the solution of Eq. (6) becomes (for details on the method see (Tesloianu *et al*., 2015))

$$\Phi=\bar{\Phi}+2a\left[\frac{E\left(s\right)}{K\left(s\right)}-1\right]+2acn^2\left[\alpha\left(\theta-\theta_0\right);s\right] \tag{8}$$

Therefore, the one-dimensional space-time dynamics of the complex fluid are given by cnoidal oscillations modes of the normalized velocity field (Figs. 1*a*-*c* and Figs. 2*a*-*f*).

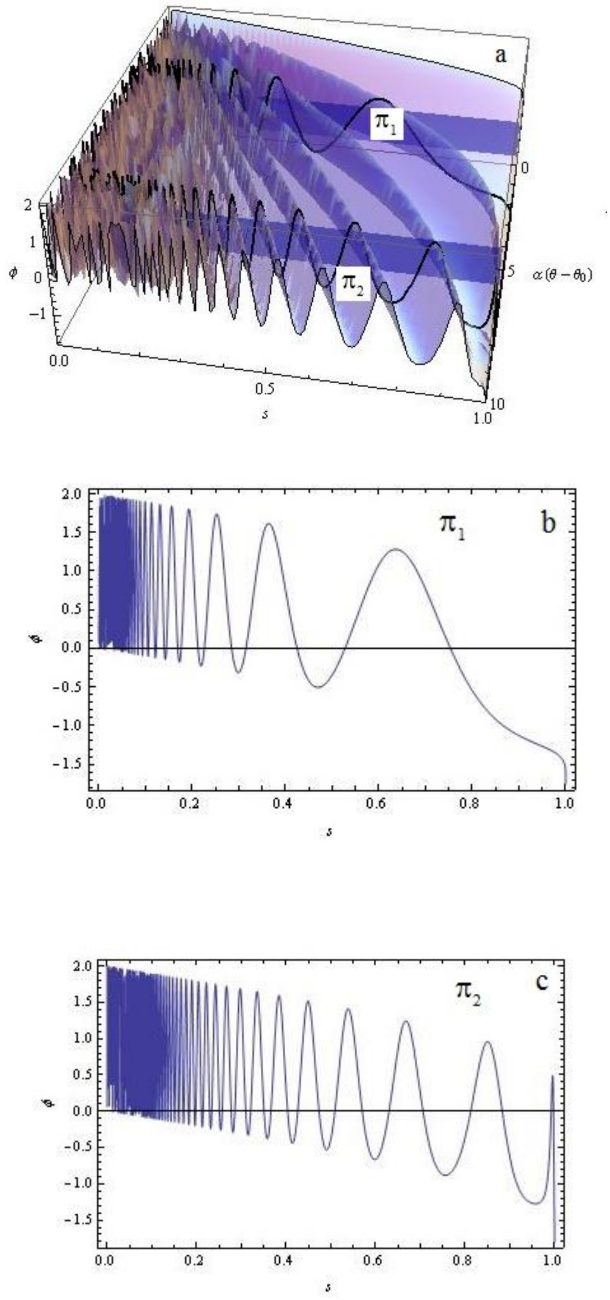

Fig. 1 – Three-dimensional (*a*) and two-dimensional (*b*, *c*) cnoidal oscillation modes of a velocity field.

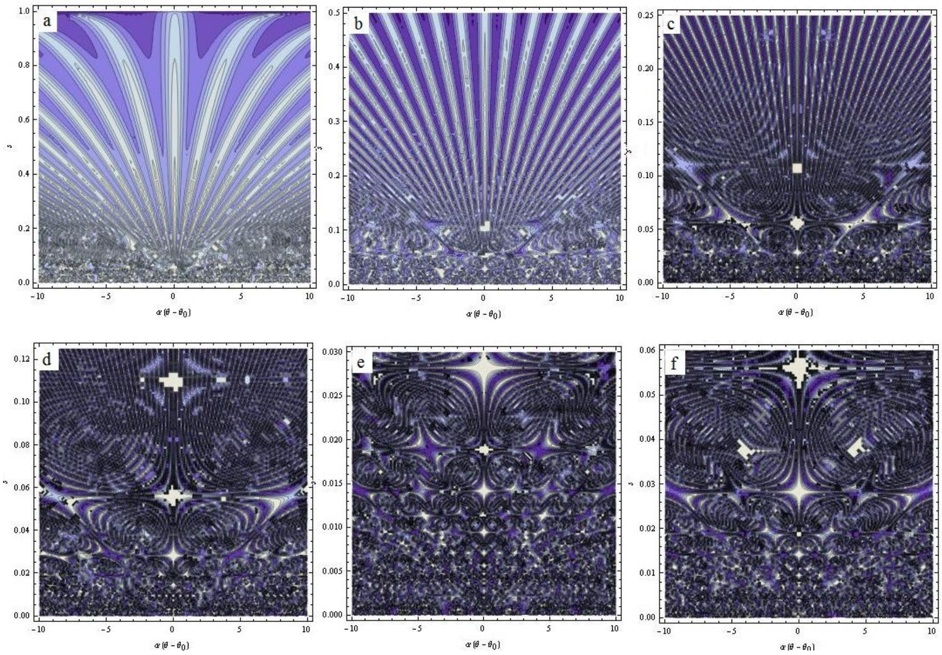

Fig. 2 – Fractal behaviors of the normalized velocity field by means of self-similarity. Contour plots for various non-linearity degrees.

In relations (7) and (8) ω is a specific pulsation, k is the inverse of a specific length, c is a specific velocity, M is the Mach number, $\bar{\Phi}$ is the average value of the normalized velocity field Φ, a is the amplitude, $K(s)$ and $E(s)$ are the complete elliptical integrals of the first and second kind of modulus s (a measure of the non-linearity degree) and cn is the Jacobi cnoidal elliptical function of modulus s (Armitage and Eberlein, 2006) and argument $\alpha(\theta-\theta_0)$ with $\theta_0 = \text{const.}$

The cnoidal oscillation modes have the following characteristic parameters:

i) Wave number

$$k = \frac{\pi a^{1/2}}{sK(s)} \tag{9}$$

ii) Phase velocity

$$U = 6\bar{\Phi} + 4a\left[\frac{3E(s)}{K(s)} - \frac{1+s^2}{s^2}\right] \tag{10}$$

iii) Quasi-period (Figs. 3*a*, *b*)

$$T = \frac{1}{\frac{3\bar{\Phi}a^{1/2}}{sK(s)} + \frac{2a^{3/2}}{sK(s)}\left[\frac{3E(s)}{K(s)} - \frac{1+s^2}{s^2}\right]} \tag{11}$$

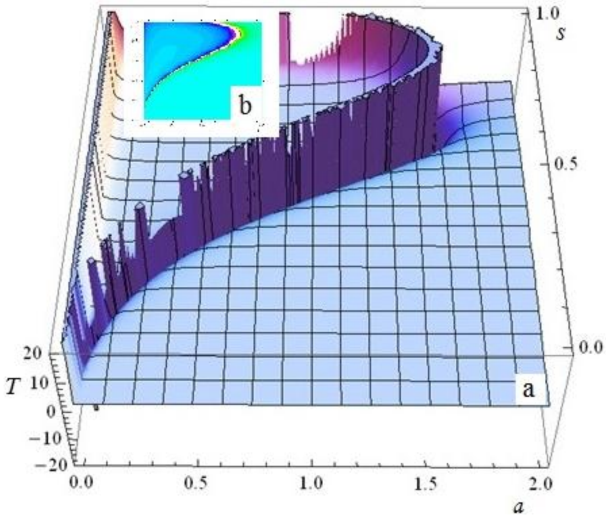

Fig. 3 – Quasi period of the cnoidal oscillation modes versus amplitude and non-linearity degree (*a*); two-dimensional contour of the quasi-period (*b*).

The "pure" sequences are obtained through the following degenerations:

i) For s→0, (8) reduces to harmonic package-type sequence (Fig. 4*a*)

$$\Phi \approx \bar{\Phi} + a + a\cos\left[k\alpha\left(\theta - \theta_0\right)\right] \tag{12}$$

characterized by wave number

$$k \approx \frac{2a^{1/2}}{s} \tag{13}$$

phase velocity

$$U \approx 6\bar{\Phi} + 8a - k^2 \tag{14}$$

and pulsation

$$\Omega = 2\pi/T \approx 6\bar{\Phi}k + 8ak - k^3 \tag{15}$$

ii) For s→1, (8) reduces to a soliton-package-type sequence (Fig. 4*b*)

$$\Phi \approx \bar{\Phi} + a_1 \operatorname{sech}^2\left[\left(\frac{a_1}{6}\right)^{1/2}\left(\theta - \theta_0\right)\right] \tag{16}$$

characterized by wave number

$$\Lambda \approx \frac{\left(2a_1\right)^{1/2}}{4k_1}, a_1 = 2a, k_1 = \frac{k}{2\pi} \tag{17}$$

phase velocity

$$U \approx 6\bar{\Phi} + 2a_1 - 12k_1 \left(a_1\right)^{1/2} \tag{18}$$

and pulsation

$$\Omega \approx 12\pi\bar{\Phi}k_1 + 4\pi a_1 k_1 - 24\pi k_1^2 \left(a_1\right)^{1/2} \tag{19}$$

For s = 0, (8) reduces to a harmonic type sequence, while for s = 1 to a soliton type one (Fig. 4*c*).

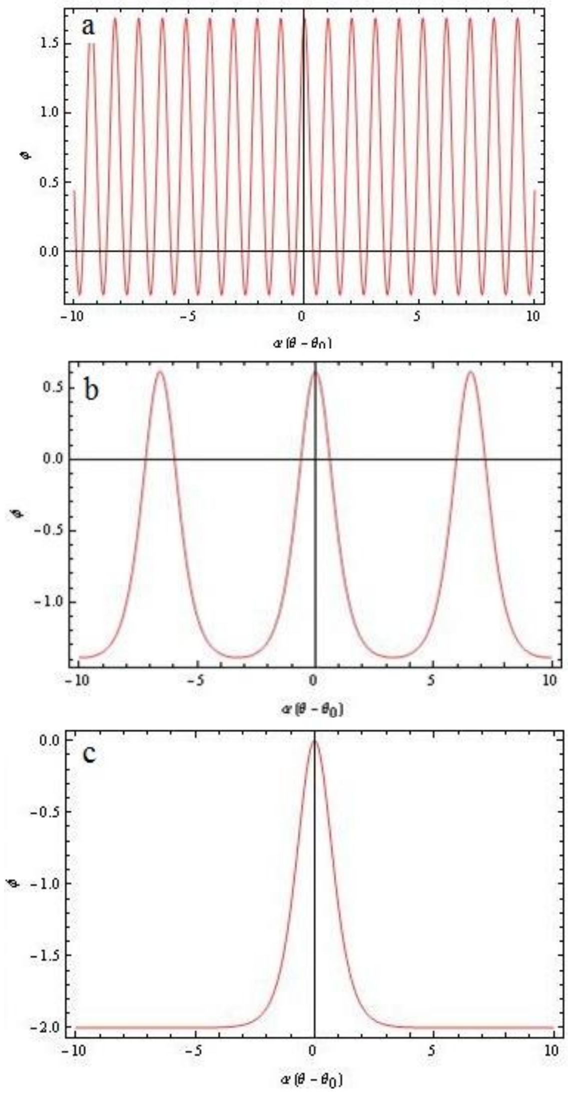

Fig. 4 – Pure sequences obtained through degenerations of cnoidal oscillations modes of velocity field: harmonic package – type sequence (*a*),
soliton package – type sequence (*b*), soliton – type sequence (*c*).

3. Results and Discussions

Natural sequences are generally presented as sequential mixtures: harmonic type sequence – harmonic package type sequence, soliton type sequence–soliton package type sequence etc. Such situations can be induced if we assume the non-linearity *s* depends on the resolution scale. The diversity of these types of oscillations can be experimentally observed (ionic oscillation period increases with the decreasing signal amplitude) in (Gurlui *et al*., 2008; Nica *et al*., 2012; Pompilian *et al*., 2012). In the case of a harmonic packet equation (11) indicates a chirping-type (Cristescu, 2008).

Eliminating amplitude *a* between (9) and (10), we obtain the following expression:

$$\left(U-6\bar{\Phi}\right)\lambda^2 = 16A(s),\ k=\frac{2\pi}{\lambda} \tag{20}$$

where

$$A(s)=3s^2K(s)E(s)-\left(1+s^2\right)K^2(s) \tag{21}$$

Nonlinearity *s* generates two distinct flow regimes of the dissipative complex fluid: non-quasi-autonomous regime (by harmonic type sequences, harmonic package type sequence or harmonic–harmonic package type sequence), and quasi-autonomous respectively (by soliton type sequences, soliton package type sequences, soliton – soliton package type sequence). The dependency *A*(*s*) – see Fig. 5, specifies that the value $s\approx 0.7$ separates the two flow regimes. For $0\le s\le 0.7$, *i.e*. in non-quasi-autonomous regime, the variable of $A(s)\approx \text{const.}$, situation in which the first relation (20) takes the form

$$\left(U-6\bar{\Phi}\right)\lambda^2 \approx const \tag{22}$$

while for $0.7\le s\le 1$, *i.e*. in a quasi-autonomous regime, relation (22) loses its validity.

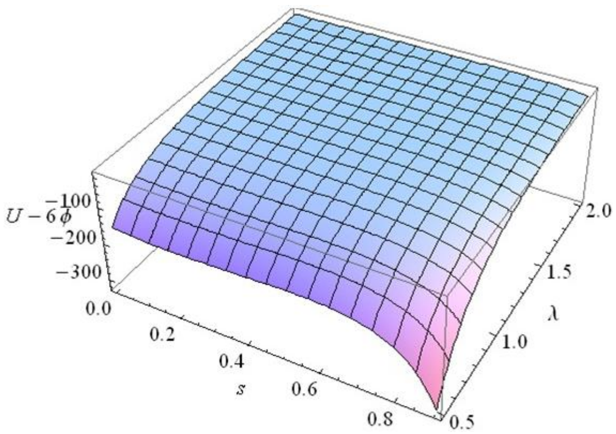

Fig. 5 – Flows regimes of the complex fluid for different non-linearity degrees.

Let us remind that one – dimensional space – time lattices of nonlinear oscillators can be associated to cnoidal oscillation modes (Toda lattices (Deift and McLaughlin, 1998)). From such a perspective, previous mentioned separating regimes can be correlated to the two sequences of the lattices spectrum (optic and acoustic).

4. Conclusions

Assuming that the particles movement of a complex fluid take place on continuous but non-differentiable curves, the geodesics equations in fractal space are obtained. These equations are identified with the stream lines of the complex fluid flows, situation in which the local self-acceleration, the self-convection, the self-dissipation and the self-dispersion of complex speed field are in balance in every point of any such line. If the dissipative effects are negligible compared to the convective and dispersive ones, its flow dynamics are given through space – time cnoidal oscillation modes of complex velocity field.

By means of space – time cnoidal oscillation modes degenerations, it results harmonic, harmonic packet, soliton, soliton packet sequences. These degenerations have been induced by different non-linearity degrees, degrees corresponding to different scale resolutions. The non-linearity degree 0.7 impose two flow regimes: non-quasi-autonomous regime, characterized by harmonic and harmonic packet sequences and quasi-autonomous regime, characterized by soliton and soliton packet sequences. We note that nature does not operate with the pure sequences mentioned above, but with mixture sequences as harmonic – harmonic packet, soliton – soliton packet etc. The self-similarity of the cnoidal modes specifies the existence of some "cloning" mechanisms (full and fractional velocity function – a function which evolves in time to a state describable as a collection of spatially distributed sub-velocity-functions that each closely reproduces the initial velocity-function shape (Aronstein and Stroud, 1997). All these show a direct connection between the fractal structure of the flow dynamics of complex fluid and holographic behaviours (Pricop *et al*., 2013).

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COMPORTAMENTE DISPERSIVE ALE FLUIDELOR COMPLEXE PRIN INTERMEDIUL NEDIFERENȚIABILITĂȚII

(Rezumat)

În această lucrare, presupunând că dinamicile entităților unui fluid complex au loc pe curbe fractale, sunt analizate efecte neliniare de tip dispersive. Rezultă că fenomenele de transport în fluidele complexe sunt dictate de moduri cnoidale de oscilație, moduri a căror degenerare implică comportamente de tip periodic, de tip cvasi-periodic, respectiv de tip soliton-kink.